COMPARISON OF METHODS OF PUMP SCHEDULING IN WATER SUPPLY SYSTEMS

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Abstract: In the domestic water supply industry, the reduction of pumping costs is a continuing objective. With the efficient scheduling of pumping operations, it is considered that 10% of the annual expenditure on energy and related costs may be saved. A typical cost function will include all of the expenditure caused by the pumping process and also consider the electrical cost of pumping taking into account the various electrical tariffs, as well as peak demand and pump switching costs. Using only fixed speed pumps, it is possible to use an efficient dynamic programming based method, provided that the storage reservoir levels are known. Other techniques that are showing fruitful results in optimisation are genetic programming and simulated annealing. This paper compares these methods and discusses which is more appropriate in this type of pump scheduling problem.

1 INTRODUCTION

A typical problem in the water supply industry is the transfer of water between interconnected reservoir systems using a least cost operation. The authors are currently engaged on a project in collaboration with the water industry to investigate cost-effective control of water transfer using fixed speed pumps, which may be either on, or off. Making certain assumptions regarding decoupling and simplification, allows us to consider the system as a source reservoir supplying a controlled reservoir via a pumping station and an equivalent pipe-line with a water demand from the controlled reservoir which is assumed to be known in advance.

The supply system considered in this paper is a section of water supply network using four fixed speed pumps to supply a reservoir from a large capacity source reservoir. The pumps may be switched at hourly intervals, and the schedule is optimised over a 24-hour period. System constraints are considered, which consist of upper and lower reservoir level constraints, and start and finish reservoir level expectations. The tariffs for day and night electricity consumption are considered, but peak tariff and switching costs are ignored for the purpose of this study. The proposal considers the pump characteristics in terms of their pumping capacities and the amount of electricity used. The scheme is used as a means of investigating a variety of techniques for the optimisation of pumping to give minimum cost.

2 PROBLEM FORMULATION

The pump-scheduling problem proposed by Mackle et al [1] is used as a starting point for setting up a model which can then be addressed for optimizing. The same problem is discussed by Savic et al [2]. The problem at this stage considers the pumping capacities of the fixed speed pumps and the amount of electricity used per hour as shown in **Table 1**. The system considered consists of one water

Table 1 *Pumping capacities of the fixed speed pumps*

Pump	Amount of water pumped in one hour [cu m]	Amount of electricity used in one hour [kWh]
Pump 1	5	12
Pump2	15	30
Pump3	25	44
Pump4	50	80

distribution reservoir, which is, supplied by four fixed speed pumps through a single water main. The optimization period is set to one day, as historic patterns of the water demand of an average day are commonly used for pump scheduling [3]. The time interval over which the electricity tariff structure is repeated is also modelled to be on 24 hours.

As is common in many electricity supply systems, the model incorporates a cheap night and more expensive day tariff. For the problem used in this paper the day-time tariff cost is set to be twice that charged during the night. The period for which the higher day-tariff applies is, for convenience, taken as being from 0800 h through to 0200 h.

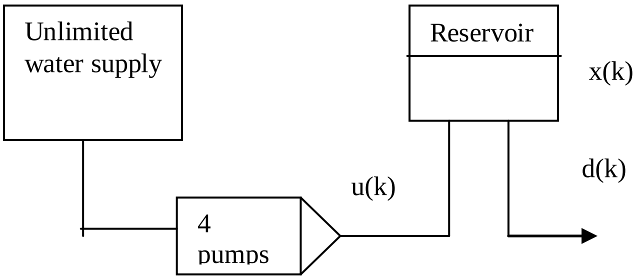

Figure 1 Water Supply System Schematic

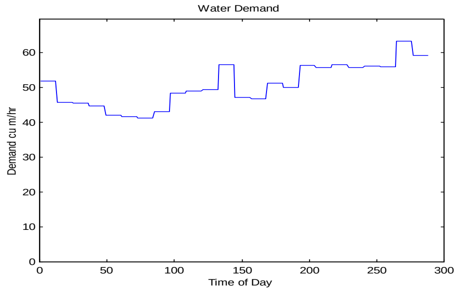

Figure 2 Water demand from reservoir

The optimization period is divided into intervals of one hour, i.e. the pumps can be either switched on or off during any hour of the day. As each of the four pumps can be run during any time interval there are 16 possible combinations of the pumps during each hour of the day. A schematic of the system is shown in **Figure 1** where:

u(k) is the total quantity of pumped water (cu m/hr)
d(k) is the water demand in cu m/hr
x(k) is the reservoir level
x(0) is initial reservoir level
The constraints are xmin < x(k) < xmax - min and max allowable levels.

The hourly demand is taken from Coulbeck [4] and is as shown in **Figure 2**. Electricity tariffs are 2.86p for peak tariff 0800 - 0200 through the day (18 hours) and 1.2p for off peak tariff 0200 - 0800 through the night (6 hours). Total reservoir capacity is 2500 cu m

3 OPTIMISATION TECHNIQUES CONSIDERED

The fundamental aim of optimisation is to establish a cost function, which is a measure of the performance of some aspect of the process under consideration. Classical methods of optimisation include linear programming, dynamic programming, hill climbing, statistical methods such as simulated annealing, and evolutionary programming methods such as genetic algorithms.

Linear and dynamic programming methods make a sequence of decisions, which together constitute an optimal policy. This approach lends itself well in principle to the pump-scheduling problem. Hill climbing and evolutionary methods generate a deterministic sequence of trial solutions based on the gradient of the cost function.

3.1 Dynamic Programming

Dynamic programming was developed by Bellman to solve problems in which a sequence of decisions is required to be made. The method lends itself ideally, in theory, to the problem of pump scheduling.
Bellmans statement of the Principle of Optimality [5] is that an optimal policy has the property that whatever the initial state and initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision.

Let x_t be the state at any time t. Let U_T be the control action at time t. In this case control of the pumps is by pumps being either on or off. Let $V_T(x_T)$ be the cost of reaching x_T at time T.
Let $g_T(x_T, y_T)$ be the cost of transition from state x_T at time T (T>t). Then the minimum principle can be expressed as:

$$\min(V_T(x_T)) = \min_{U_T}[g_T(x_T, x_t) + \min(V_t(x_t))]$$

i.e. the minimum of (the cost of transition from x_t to x_T + minimum of getting to the point x_t)

The minimum principle holds for any form of cost function and whatever the effect of the controls on the system. Dynamic programming is, in effect, the repeated application of the minimum principle at a sequence of intervals. In the pump-scheduling problem considered, the state of the system is taken as the volume of water in the reservoir. In order to calculate the minimum cost, the reservoir is divided into discrete levels. The stages are taken as the hourly decisions made over the 24-hour period. [6].

In order to give sufficient accuracy in the calculation, it is necessary to divide the reservoir into about 50 levels. Using four pumps with therefore 16 combinations, the number of possible transitions at each stage is 16x50. For hourly decisions over the 24-hour period, the cost must therefore be stored at 16x50x24. Clearly as more reservoirs and more pumps are considered, the time and storage requirements of dynamic programming become very large.

3.1.1 Optimisation of Pumping using Dynamic Programming

The classical approach for optimal scheduling of water pumping is:

Minimise{pumping cost+treatment cost}
Subject to:
Water network equations, pressure constraints at critical nodes, flow constraints in critical pipes, reservoir level constraints [7]. The resulting controlled reservoir level over the 24-hour period is shown in **Figure 3** with the pump schedule in **Figure 4**.

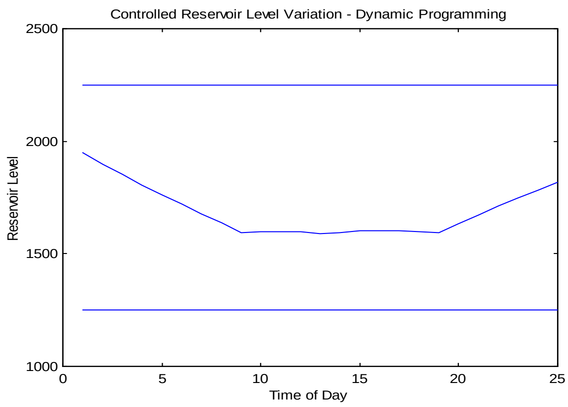

Figure 3 Reservoir level with optimal scheduling of pumping using dynamic programming

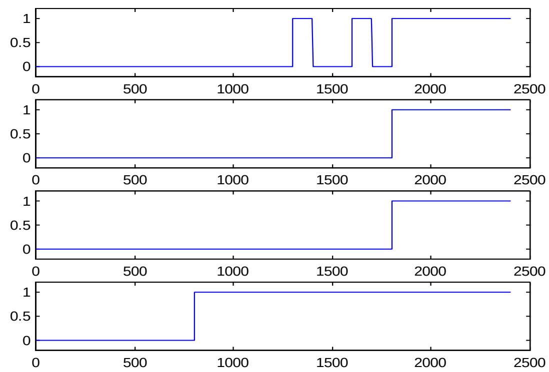

Figure 4 Pumps ON or OFF during 24 hour period P1, P2, P3 and P4

3.2 Simulated Annealing

The heart of the method of simulated annealing is an analogy with thermodynamics called annealing. This is the process by which liquids freeze and crystallise, or metals cool and anneal. At high temperatures, the molecules of a liquid move freely with respect to one another. As the liquid is slowly cooled, thermal mobility reduces. The atoms line themselves up and form a pure crystal which is completely ordered. The crystal is the state of minimum energy of the system. At each temperature during the annealing process, slow cooling enables the system to achieve equilibrium. If the temperature is lowered too quickly, the system does not have sufficient time to achieve equilibrium, and the resulting configuration might have many defects in the form of high-energy, metastable, locally optimal structures. Natures minimisation algorithm is based on the Boltzmann probability distribution,

prob(E) ~ exp(-E/kT)

This describes the process by which a system in thermal equilibrium at a temperature T has its energy probabilistically distributed among all different energy states E. Even at low temperatures, there is a small chance of a system being in a high energy state. This means that there is a corresponding chance for the system to get out of a local minimum energy state in favor of finding a better, more global, minimum. The quantity k is Boltzmann's constant and is a constant of nature, which relates temperature to energy.

The original incorporation of these principles into numerical calculations was first carried out by Metropolis [8]. Consider a system which is assumed to change its configuration from energy E1 to energy E2 with probability $p = \exp[-(E1-E2)/kT]$. If $E2 < E1$ then the system is arbitrarily assigned a probability p=1, and the system will always take this option (it is a reduction in energy). If however $E2 > E1$ then the scheme may take this option with a probability p. The scheme is one, which goes in the general direction of reducing energy, but sometimes allowing an energy increase.

The requirements of a Metropolis algorithm are:

1 A description of the possible system configurations
2 A means of generating random changes in the configuration. These are the 'options' presented to the system
3 An objective function E, which is an analogue of the energy. The goal of the procedure is the minimisation of this function
4 A control parameter T which is the analogue of temperature. The temperature is lowered at each successive pass through the algorithm using an annealing schedule.

3.2.1 Optimisation of Pumping using Simulated Annealing

As a problem in simulated annealing, the pump optimisation problem is handled as follows:

1 Configuration. The four pumps may be either on or off. This may be represented as a 0 or a 1. Since we are considering the state of these four pumps on an hourly basis over 24 hours we have 4X24 = 96 possibilities.
2 Random Rearrangements. Two useful rearrangements suggested by Lin [9] are (a) to remove a section of the possibles and replace them in the opposite order and (b) to remove a section and replace in a different position.
3 Objective Function. The cost function in the case considered is a combination of the electricity cost together with the penalties for not meeting the end reservoir level requirement and for exceeding the upper and lower level limits. E = Sum of Elec costs at various tariffs + sum of exceed limits + final error
4 Annealing Schedule. This is arrived at by experimentation. It may be appropriate to choose some random rearrangements and use them to determine the range of values of deltaE that will be obtained. This then allows the choice of T with starting value considerably larger than deltaE, and then gradually reducing T in multiplicative steps until either the limiting number of temperature steps has been reached, or the number of successful reconfigurations has reduced to zero [10][11]. The results of the scheduling optimisation using simulated annealing are shown in **Figure 5**

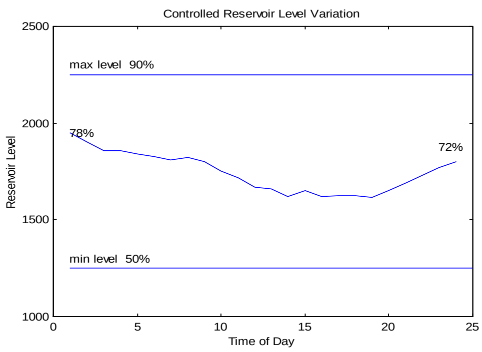

Figure 5 Reservoir level - results of the scheduling optimisation using simulated annealing

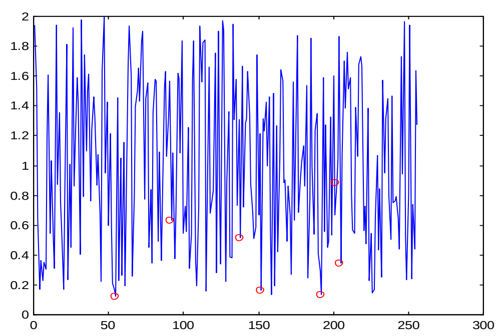

Figure 6 Minimum estimation problem similar to pump optimisation scheme

3.3 Genetic Algorithms

Genetic algorithms (GAs) use a stochastic global search method, which mimics natural genetic evolution. They operate on a population of potential solutions by applying the principle of 'survival of the fittest' to produce increasingly better approximations to the solution. At each generation, a new set of approximations is generated by the process of selecting individuals according to their level of fitness of the cost function in the problem in which they are being used. This leads to the creation of a set of individuals better suited to the environment in which they exist than the individuals from which they were created. To progress from one population to the next, members of the current population may be modified using reproduction, crossover, mutation and inversion.

A string of numbers is often used to represent the decision variables and the possible choices. In the case of the pumping station considered in this investigation it is convenient to represent each of the four pumps by a 1 or a 0 as to whether it is on or off. This leads to a convenient representation of the population over a 24-hour period by a binary string word length of 4 x 24 (i.e. 96).

The overall cost function used with the pumping schedule includes the total electricity cost of pumping over the 24 hour period together with penalty costs associated with violating constraints such as exceeding reservoir levels and not meeting final level requirements.

3.3.1 Optimisation of Pumping using Genetic Algorithms

The GA search first generates a family of initial population using random seeding. A fitness function is then used to assess the performance of each of the individual members of the population. A proportion of these are then chosen according to their relative fitness and recombined to produce the next generation.

Genetic operators such as recombination and mutation are used to manipulate the chromosomes on the assumption that certain individual's genes produce, on average, fitter individuals. Finally the objective function is evaluated, a fitness value assigned to each individual, and the individuals selected again for mating according to their fitness.

The process continues through subsequent generations until either a certain number of generations have been completed or a mean deviation in the population has been achieved [12][13][14][15].

A typical cost function graph for a range of chromosomes is shown in **Figure 6** with successive estimates of minimum shown as circles. This figure shows the seeming randomness of the problem with a problem such as pump scheduling. The corresponding minimisation is shown in **Figure 7**. The reservoir level with GA pump optimisation is shown in **Figure 8**.

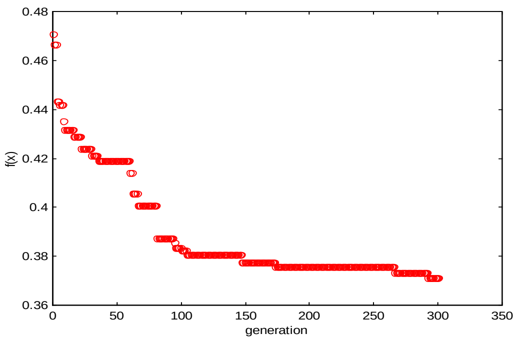

Figure 7 Cost function minimisation

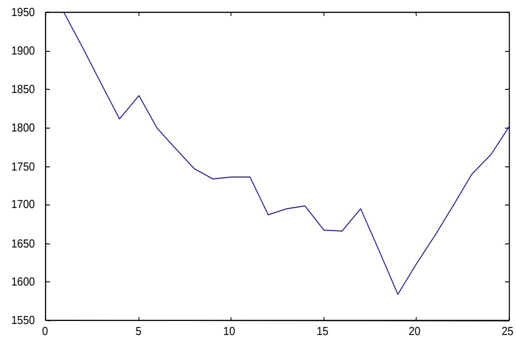

Figure 8 Reservoir level using genetic algorithm pump optimisation

4 CONCLUSIONS

In comparing the three different approaches to optimisation, dynamic programming is generally faster than the others, but has the problem that as the complexity of the system increases, the size of the storage requirements becomes excessive. It is for example not appropriate for pump optimisation systems with more than two reservoirs. Simulated annealing provides a sub-optimal result taking about 10 minutes to run with the pump optimisation problem. The development of an appropriate cost function is worth some consideration, and may lead to a better optimal result. Genetic algorithms are easy to use, but again it is the cost function, which presents the problem. The algorithm took approximately 20 minutes to run in this case.

5 REFERENCES

1 Mackle G et al Application of Genetic Algorithms to Pump Scheduling of Water Suppl*y Int Conf. On Genetic Algorithms in Engineering Systems* pp 400-405 1995

2 Savic D A et al Multiobjective Genetic Algorithms for Pump Scheduling in Water Suppl*y Evolutionary Computing AISB Int Wkshp* Selected Papers Springer-Varlag 1997

3 Coulbeck and Orr Development of an Interactive Pump Scheduling Program for Optimised Control of Bulk Water Supply *Proc Int Conf on Systems* Eng pp 176-186 1981

4 Coulbeck B and Orr C H [1984] Optimized pumping in water supply systems *IFAC 9th Triennial World Congress*, Budapest, Hungary

5 Bellman R. E. Dynamic Programming *Princeton University Press* 1957

6 Technical Report TR232 *Water Research Centre* Swindon 1985

7 Brdys & Ulanicki Operational Control of Water Systems *Prentice Hall*

8 Metropolis N, Rosenbluth A, Rosenbluth M, Teller A and Teller E Equation of State Calculation by Fast Computing Machines *J Chem Phys*., Vol 21 p 1087 1953

9 Lin S *Bell System Technical Journal Vol 44 p2245* 1965

10 Press W H, Flannery B P, Teukolsky S A, Vetterling W T Numerical Recipes – The Art of Scientific Computing *CUP* 1990

11Sousa J et al On the Quality of a Simulated Annealing Algorithm for Water Network Optimisation Problems *Water Industries Systems – Modelling and Optimisation Applications* , Ed Savic D A and

12 Savic, D.A., Waiters, G.A. and Schwab, M. (1997). Multiobjective genetic algorithms for pump scheduling in water supply, *Evolutionary Computing workshop*, AISB '97, Manchester, 7-8 April.

13 Simpson, A.R., Dandy, G.C. and Murphy, L.J. (1994). Genetic algorithms compared to other techniques for pipe optimisation, *Journal of Water Resources Planning and Management,* ASCE, 120 (4), July/August, 423-443.

14 Dandy, G.C., Simpson, A.R. and Murphy, L.J. (1996). An improved genetic algorithm for pipe network optimisation. *Water Resources Research, Vol.* 32, No. 2, Feb., 449-458.

15 Mackle, G., Savic, D.A. and Waiters, G.A. (1995). Application of Genetic Algorithms to Pump Scheduling for Water Supply, *Conference on Genetic Algorithms in Engineering Systems: Innovations and Applications,* GALESIA `95, IEE Conference Publication No. 414, Sheffield, UK, pp. 400-405.